TOPIC : TESTS FOR V. SUBSP.

Quest: $V = \{0\}$; $F = \mathbb{R}$. $V(F)$ is a V.F.
How many subspaces of $V(F)$.
1) One.

TEST ① Let $V(F)$ be a V.S over the field F &
$W \subseteq V$. Then.
① $0 \in W$ ② $x, y \in W$; $x + y \in W$
③ $\alpha \in F$, $x \in W$; $\alpha x \in W$.

TEST ② Let $V(F)$ be V.S. over F. And W is a non-empty subset of V. Then W is said to be a vector subsp. of $V(F)$ if
1) $\alpha x + \beta y \in W$ $\forall$ ~~α, β~~ $x, y \in W$, $\alpha, \beta \in F$.

i.e. $\boxed{\alpha x + y \in W}$

Quest. $V(F) = F^n(F)$ be a V.S. where $(F, +, \cdot)$ be a field.

1. $W = \{(a_1, a_2, \ldots, a_n) \in V ; a_1^2 + a_2^2 + a_3^2 \leq 1\}$

$x\alpha + y = (x_1, x_2, \ldots, x_n)\alpha + (y_1, y_2, \ldots, y_n)$
$= (x_1\alpha + y_1, x_2\alpha + y_2, \ldots, x_n\alpha + y_n)$

$(x_1\alpha + y_1)^2 + (x_2\alpha + y_2)^2 + (x_3\alpha + y_3)^2$
$= (x_1^2 + x_2^2 + x_3^2)\alpha^2 + (y_1^2 + y_2^2 + y_3^2) + 2\alpha(x_1 y_1 + x_2 y_2 + x_3 y_3)$
$\nleq 1$
$\therefore$ not a subsp.